may 2, 16

Talk MIT:

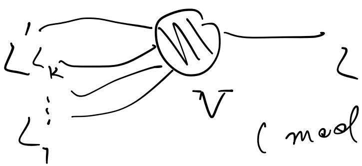
work in progress w. Paul Biran (M,ω) fixed

I "Theorem". There is a Lagrangian cob. category

$\mathcal{C}^* = \mathcal{C}ob^*_{Lag}(M)$

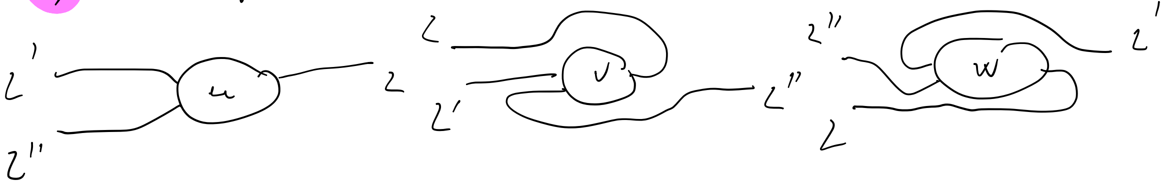
a. with objects $L \in \mathcal{L}ag^*(M)$ = class of Lagr. submflds (possibly immersed) of M subject to $*$,

b. with morphisms Lagr. cobordisms $V \in \mathcal{L}ag^*(\mathbb{C}\times M)$ (possibly immersed)



; $V : L \longrightarrow L'$ (mod horizontal isotopy).

c) with special triangles (w, v, u):



d) a relation $\sim$ on mor $\mathcal{C}^*$

$v \sim v' \iff$ 

$\in \mathcal{L}og^*(\mathbb{C} \times M)$

so that for an appropriate choice of $*$, $\sim$ is an equivalence and the quotient category $\hat{\mathcal{C}}^* = \hat{\mathcal{C}}ob^*_{\mathcal{L}og}(M)$ has the properties:

i) $\hat{\mathcal{C}}^*$ is triangulated with triangles induced from the special ones;

ii) the subcategory generated by the embedded $\mathcal{L}ag$ is $\simeq DFuk^*(M)$

iii) each exact triangle is $\sim$ to a surgery exact triangle.

iv) $\hat{\mathcal{C}}^* \cong Dem(\mathcal{L}og^*(M))$

Explain " — ": in progress etc.

Remarks:

a) All objects of $DFuk^{*}(M)$ have representatives through immersed Lagrangians. All exact triangles also have representatives as strongly exact triangles.

b) $*$ should be thought off as "unobstructed"

$Log^{*}(-)$, $Log^{*}(\mathbb{C} \times -)$ behave like sheaves if $*$ is general enough so the key to go from local $\longrightarrow$ global is to see whether $*$ is preserved.

c) immersed Lagrangian Floer theory is tricky: "unobstructed" is highly dependent on J a.c structure. So in reality we should use pairs $(L, J), \dots (V, J)$ etc. It's very closely related to Legendrian Contact Hlgy.

d) In * can be included additional constraints typical for cobordism: spin structures, grading etc but also idempotents (non-trivial).

e) Associated cobordism group: $\Omega^*_{Lag}(M) \cong K_0(D\mathrm{Fuk}^*(M))$.

f) Other workers: Seidel, Biran-C, Akaho, Akaho-Joyce, Fooe, Allston, Allston-Bao, useful discussions with Seidel and Abouzaid.

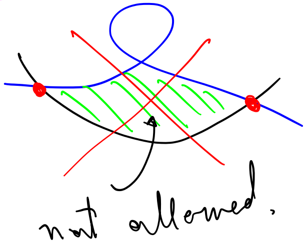
Light remarks: Statement is not quite obvious:

- $\hat{\mathcal{C}}$ = additive category ($\mathrm{mor}_{\hat{\mathcal{C}}}(L, L')$ are groups)?
- constructions of cobordisms: at least if we admit immersed, surgery suffices.

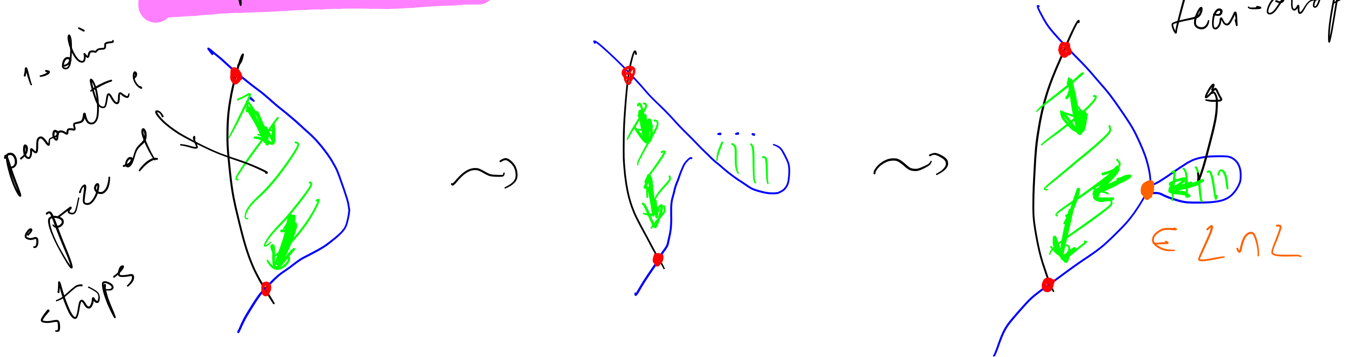
I' Key step in the construction.

a. L immersed, N embedded

to define $CF(N, L)$ do as usual + count strips that do not jump branches):



Obstruction to $d^2 = 0$ are tear drops through the self intersection points of L



6. Key new observation that is at the core of today's theorem:

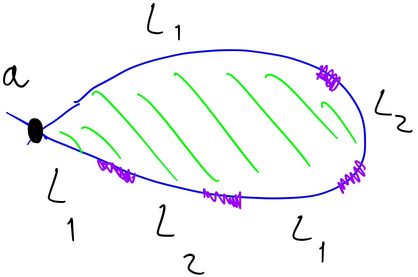
Assume L_1, L_2 embedded and let $c \in CF(L_2, L_1)$ be such that $dc = 0$. We work over $\mathbb{Z}/2$. Thus:

$$c = \sum_{i=1}^{k} a_i \,, \quad a_i \in L_1 \cap L_2 .$$

Let $L = L_1 \#_c L_2$ be the surgery of L_1 & L_2 at the points $a_1, \dots, a_k$. Clearly L is, in general, immersed.

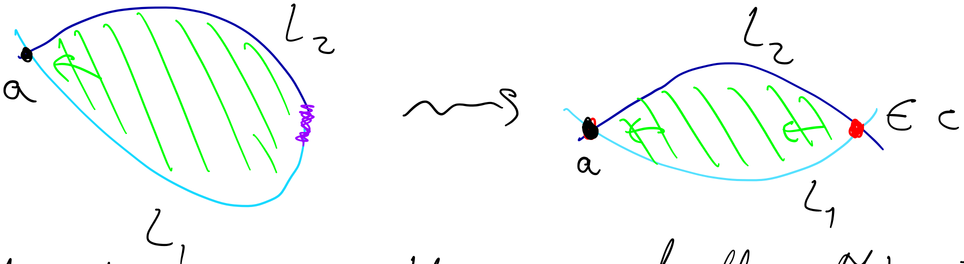
<u>Lemma</u>: L, V unobstructed. (for a particular J, and a very small surgery handle H_ε)

Need to show $\#_2$ tear drops through pt $a \in L_1 \cap L_2 = 0$.



assuming regularity + index arguments may reduce to a single switch.

from L_2 to L_1:



The switch takes place on the sergey handle. Make that very small and degenerate it to an intersection pt.

$\#_2$ tear-drops through $a = \langle dc, a \rangle = 0$. !!

Remark : Subtleties :

a) The almost complex structure is fixed. If one changes it need "bounding co-cycles". That is where the μ^k's are hiding!

b) The cobordism V does not have only double pts.

c) Iterate the process leads to "genealogy problems".

III Many interesting ingredients but will talk about one at your choice:

A. The eq relation ~: How does it lead to an abelian category.

B. Idempotents: how are they integrated in the picture

C. How does the Dehn twist fit in the picture.

public favorite!

D. How to include a morphism in an exact tr. and how to show that the relevant cat is triangulated,

E. Regularity: how does one set up the correct moduli spaces and get regularity.