Where only mathematicians boldly go...

THE GUARDIAN
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For more than 300 years Fermat's Last Theorem has tantalised scholars and amateurs alike. Yesterday a shy British professor quietly delivered the answer

The number's up for maths' greatest riddle

Andrew Granville and Ian Katz

IN THE EVENT the 40-year-old mathemetician strung out the suspense to the last. He had spoken for over two-and-a-half hours and covered the four blackboards in the lecture theatre several times before he turned to one of them a little after 10.30am yesterday and wrote a few lines of algebraic script.

By then most of the 100 or so scholars gathered at the Isaac Newton Institute in Cambridge did not need to see them to realise what he was claiming but the effect was nevertheless electrifying. "He just said 'I will stop here'," said Enrico Bombieri, a professor of mathematics at Princeton University and winner of the coveted Fields Medal, the mathematical equivalent of the Nobel Prize. "There were a few seconds of silence and then the audience burst into tremendous applause as the truth started seeping in."

The truth was that perhaps the most enduring and perplexing unsolved problem in mathematics had finally submitted to an ingenious assault by the Princeton-based British mathematician. At the end of the three-part lecture, unprepossessingly entitled "Modular forms, elliptic curves and Galois representations", he calmly announced that he had proved Fermat's Last Theorem.

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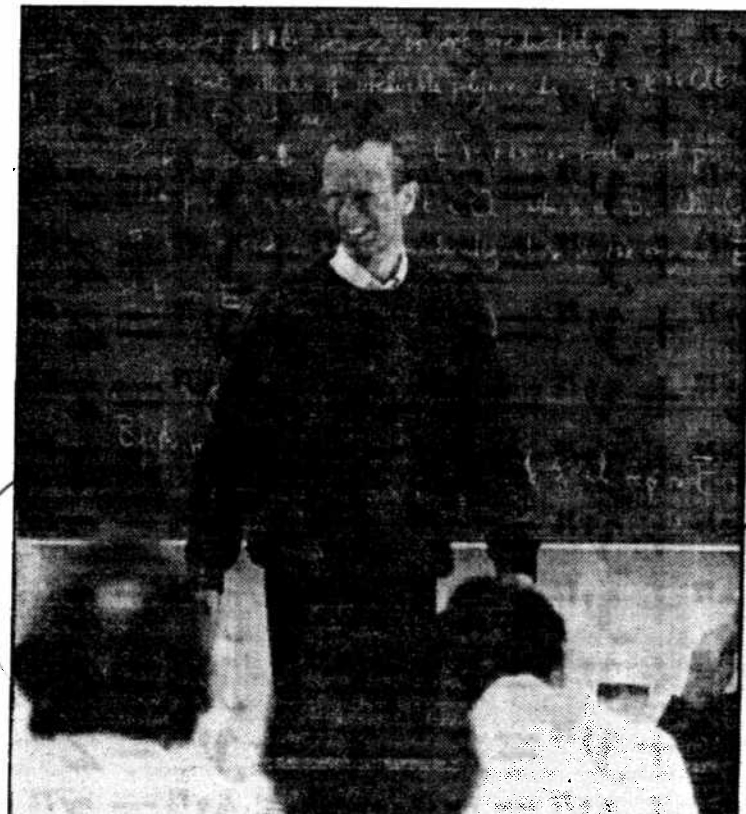

Summing up . . . Professor Wiles enters the mathematical maze

Fermat's Theorem for beginners

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others. Fermat, a jurist from Toulouse, studied mathematics as a hobby. He did not formally publish his work but rather disseminated his ideas in letters, challenging others to match and/or admire his understanding.

Fermat was evidently inspired by the Arithmetic and made many notes in the margin of his copy. After his death, his son Samuel published these notes and among them was the following tantalising comment, beside the description of Pythagoras's Theorem: ". . . it is impossible for a cube to be written as a sum of two cubes or a fourth power to be written as a sum of two fourth powers or, in general, for any number which is a power greater than the second to be written as a sum of two like powers. I have a truly marvellous demonstration of this proposition which this margin is too narrow to contain."

The assertion amounted to a claim about equations of the same type as Pythagoras's Theorem. In mathematical notation it stated that one cannot find whole numbers x, y, z and n, where n is bigger than 2, for which $z^n = x^n + y^n$. Whether Fermat was being overly optimistic about his "marvellous demonstration" we shall probably never know — it was never found — but his argument has not been reproduced in the intervening three-and-a-half centuries, despite no shortage of effort to do so.

His seductively simple assertion is known as Fermat's Last Theorem. Substantial prizes have been offered for its solution (the largest, the Wolfskhel Prize, was rendered trivial by the German hyper-inflation of the 1920s). The great mathemati-

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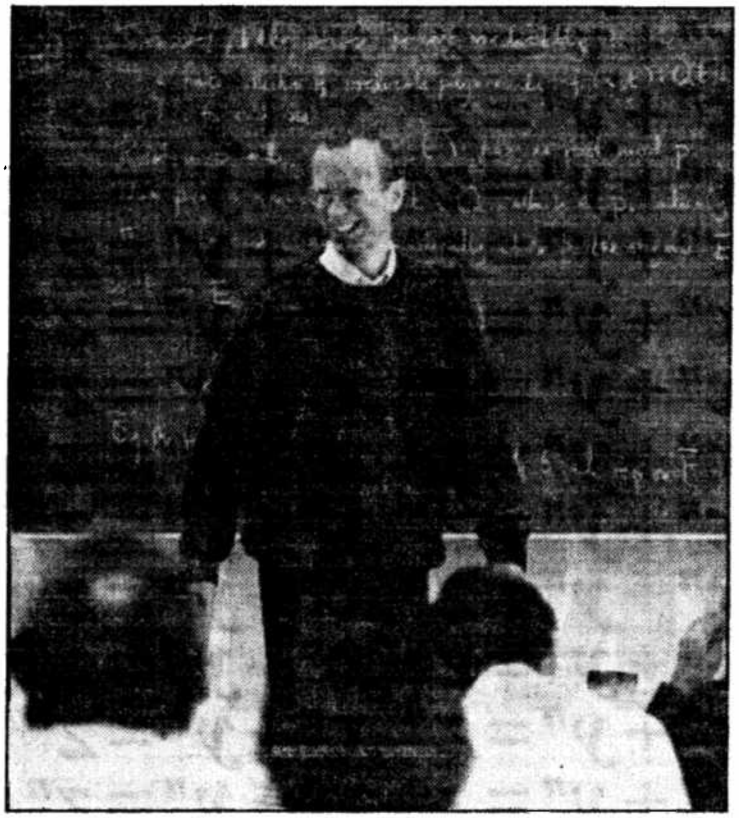
Of all scholars, mathematicians are perhaps the least given to shows of emotion about their work. They know too well that what looks like a theory today can be reduced to a handful of moderately good ideas under the close analysis of a rival the next day. But the mood at the institute, opened a little under a year ago to conduct brainstorming sessions on major mathematical problems, was triumphant. At a small reception held after the lecture, Peter Goddard, the institute's deputy director, broke out a few bottles of champagne.

Within hours the university had issued a comprehensive statement including a beginner's guide to Fermat's Last Theorem. The press release was subtitled (with less exaggeration than most exercises in public relations) "Mathematical result of the century".

Fermat's Last Theorem lies in the rarified territory of Number Theory, on the face of it one of the most recondite and other-worldly provinces of mathematics. The agenda for the week-long conference at which Wiles was speaking offers few toe-holds to the layman. Yesterday's list, for example, announced that U Jannsen would be discussing "rigidity of K-cohomology". Today at 11.30am W Messing will deliver some remarks on "the f-ness of the image of the Abel-Jacoli map".

Although Wiles's lectures appear under a similarly inaccessible title, the real appeal of Fermat's Last



Summing up . . . Professor Wiles enters the mathematical maze

Fermat's Theorem for beginners

The arithmetical equation $9+16=25$ could also be written as $3^2+4^2=5^2$. Similarly, $25+144=169$ could be written as $5^2+12^2=13^2$.

In general terms, both of these could be expressed as $x^n+y^n=z^n$.

Fermat's Last Theorem says if *n* is a whole number larger than two, and *x, y* and *z* are all whole numbers larger than zero, the equation has no solution.

For example, if $n=3$, it is impossible to find any whole numbers for *x, y* and *z* which would fit the equation $x^3+y^3=z^3$.

Experiments by trial and error, using various values for *n* suggest that Fermat's Last Theorem is *probably* true. But the number of possible values for *n* is infinite, which is one reason why *proving* the truth of the theorem defeated mathematicians for so long.

Theorem lies in its apparent simplicity. Its origins can be traced back to ancient Greece and that schoolboy's staple, Pythagoras's Theorem. That is, in any triangle which contains a right-angle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides. (In mathematical notation, $z^2=x^2+y^2$, where *z* is the length of the hypotenuse and *x* and *y* are the lengths of the other two sides.)

This rule allowed the ancient Greeks to construct an accurate right-angle. The idea was simply to cut pieces of rope of lengths *x, y* and *z* units, pull the ropes taut (holding the ends together) and, bingo, a right-angle. For this to be practical they needed *x, y* and z to each be whole numbers; and the example $5^2=3^2+4^2$ sufficed for their purposes. Other examples that they recorded included $13^2=5^2+12^2$ and even $8161^2=4961^2+6480^2$.

One of the great intellectual masterpieces of the ancient Greek world was Diophantus's Arithmetic. This work, available in Latin translation in the 17th century, was an important inspiration for the scientific renaissance of that period, read by Fermat, Descartes, Newton and others. Fermat, a jurist from Toulouse, studied mathematics as a hobby. He did not formally publish his work but rather disseminated his ideas in letters, challenging others to match and/or admire his understanding.

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His seductively simple assertion is known as Fermat's Last Theorem. Substantial prizes have been offered for its solution (the largest, the Wolfskhel Prize, was rendered trivial by the German hyper-inflation of the 1920s). The great mathematicians of history have been in two minds as to whether to work on it. Ernst Kummer, the German mathematician of the last century who did so much to establish modern algebra, wrote that Fermat's Last Theorem is "more of a joke than a pinnacle of science", yet his most important work originated in his failed attempts to prove it. Fermat's Last Theorem is the question mathematicians love to hate: it is really only one example of an equation, and then not particularly relevant in a general study of equations. On the other hand, there is no denying its charm and simplicity. "It's not a crucial result of the kind that people have been using for years without being able to prove," Judith Fields, a historian of mathematics at Imperial College, London, says. "But a lot of Number Theory theorems have a rather beautiful simplicity about them and this is a particularly simple one. It looks as if you ought to be able to prove it, damn it."

The theory's simplicity meant that as many amateurs as professionals were attracted to the challenge of providing an unassailable proof. Every few years for most of the last century the newspapers have reported yet another purported proof, which is subsequently found to be lacking in some way. Until the

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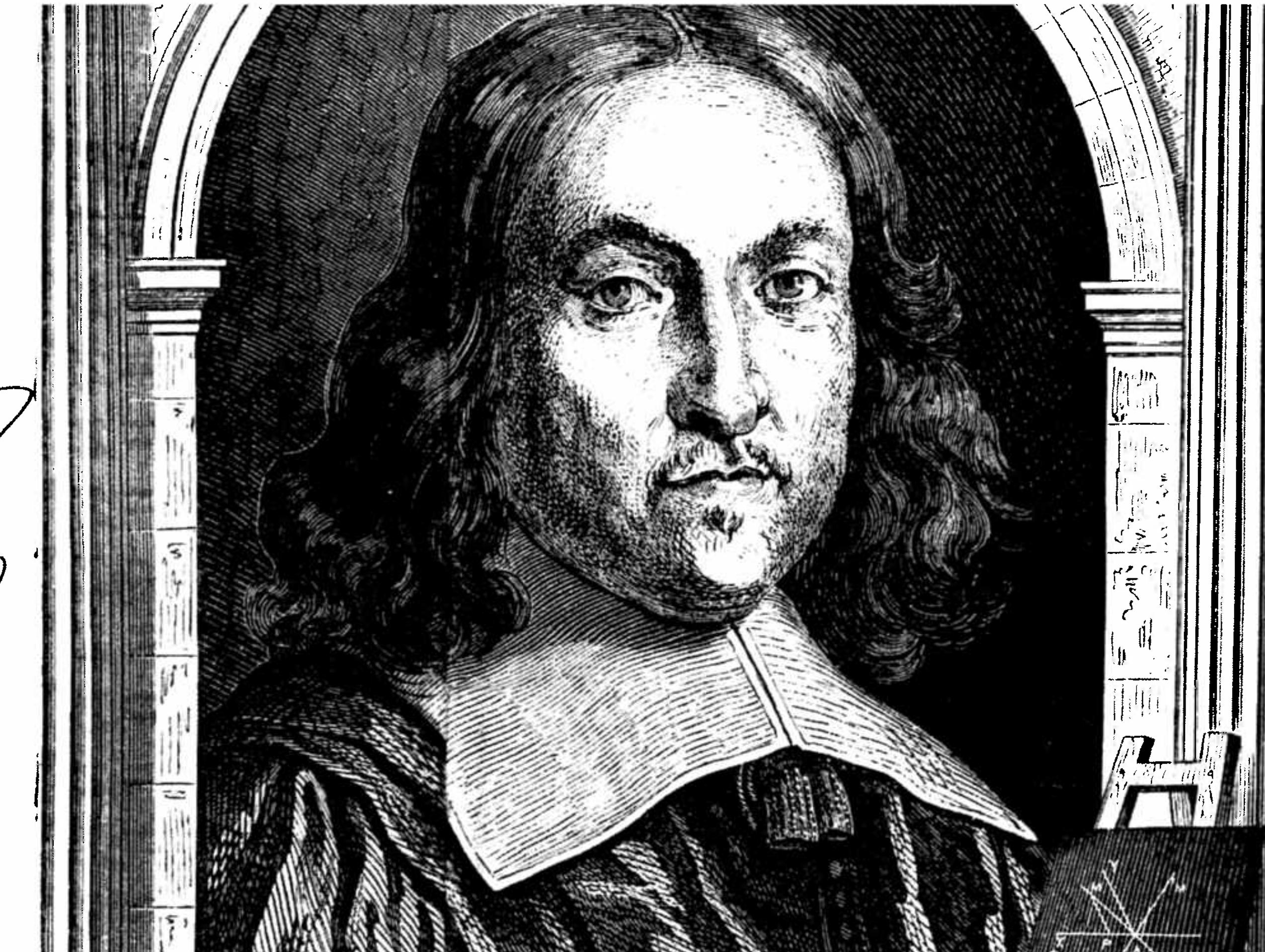
last decade the question seemed unassailable. To be sure, many interesting approaches have been proposed, persuading us of the truth of Fermat's Last Theorem, but without providing a proof that can be checked in every case. One such "proof" is known to work for every exponent n up to 4,000,000: that is, for every number n up to four million there are no whole numbers x, y and z for which $z^n = x^n + y^n$ has solutions. However this method of proof involves an individual computer verification for each n, and so will not give the result for every possible value of n. In 1983, a young German mathematician, Gerd Faltings, took another great step toward a proof, showing that for any given n greater than two there is only a finite number of whole number solutions to $z^n = x^n + y^n$: however, it seems unlikely that his methods can be modified to actually show that there are no solutions.

Wiles's approach to Fermat's Last Theorem comes from a somewhat different direction, with rather different types of ideas. It all began in 1955, with a question posed by the Japanese mathematician Yutaka Taniyama: Could one explain the properties of elliptic curves, equations of the form $y^2 = x^3 + ax + b$ with a and b given whole numbers in terms of a few well-chosen curves. That is, is there some very special class of equations that in some way encapsulate everything there is to know about our elliptic curves? More work by Andre Weil, brother of the philosopher Simone Weil and Goro Shimura, a Japanese mathematician produced an esoteric answer dubbed the Shimura-Taniyama-Weil conjecture. There the



Pierre de Fermat . . . 'A truly marvellous demonstration'

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matter stood until 1986 when Gerhard Frey from Saarbrücken made the most surprising and innovative link between this very abstract conjecture and Fermat's Last Theorem. When it had been reinforced by the work of two other mathematicians, it showed that a counter-example to Fermat's Last Theorem would directly contradict the Shimura-Taniyama-Weil conjecture.

Enter Andrew Wiles. Slightly-built and bespectacled, he had studied mathematics at Oxford before moving to Cambridge to work on a PhD. He quickly impressed his supervisor, Professor John Coates. "He did wonderful work as a research student and proved a very spectacular result in his thesis." He was offered a research fellowship at Clare College but within a year had been lured to Harvard, where he continued his research and taught until moving to Princeton as a full professor in the early 1980s. A few years later he was awarded a Royal Society research professorship but he was quickly sucked back down the brain drain, returning to Princeton.

His reputation as one of the deepest pure mathematicians of his generation had already been secured by a number of important breakthroughs, including his part in proving a theory known as The Main Conjecture of Iwasawa. This week he showed that he has proved the Shimura-Taniyama-Weil conjecture for an important class of examples, including those relevant to proving Fermat's Last Theorem. His work can be viewed as a blend of arithmetic and geometry, and has its origins back in Diophantus's Arithmetic. However he employs the latest ideas from a score of different fields using concepts credited to mathematicians from Britain, France, Germany, Italy, Japan, the United States, Canada, Russia and Colombia. Few suspected that their work might have this kind of application.

THERE are probably no more than half a dozen people in the world capable of fully understanding all the details of what Wiles has done, all but one of them present at the Isaac Newton Institute this week. Some had guessed the implication of his reasoning as he began to outline his theory in his one-hour talks on Monday and Tuesday. But Wiles gave precious little away. Until the last moment not even his closest associates knew how much he would claim in his final lecture.

"He's an extremely careful mathematician who does not make rash statements," Bombieri says. "The conclusion is the crown jewel of a theory but in his last talk he still needed to perform some difficult tricks, little miracles even."

Given the enormous complexity of his work it could take several months to be certain that every detail Wiles relies upon, from such an array of areas, is correct.

Yesterday however, some of the world's leading mathematicians were toasting Wileswith unusual gusto. "It is a beautiful and not too difficult argument," Bombieri says. "I have to admit that some parts of the proof I can follow and others I have a more superficial understanding of but the whole structure of the proof is very tight and very solid."

But as the global mathematical grapevine buzzed with the news from Cambridge there were a few sighs of disappointment. At approximately 1,000 pages long Wiles's proof was never going to fit into the margin of any text book. As one scholar put it: "Many of us, while hailing Wiles's magnificent achievement, yearn for Fermat to have been correct ... for the truly marvellous, and presumably comparatively straightforward, proof to be recovered."

Andrew Granville is associate professor of mathematics at the University of Georgia