

MSI: THE ANATOMY OF INTEGERS AND PERMUTATIONS

by JENNIFER AND ANDREW GRANVILLE

“*Mathematical Sciences Investigation (MSI): The Anatomy of Integers and Permutations*”
is a brand new experimental work that blurs the boundaries between pure mathematics,
film, and live performance.

CAST

Actors/Readers (in order of appearance):

Narrator Lorraine Wochna
Jack Von Neumann Carl Wallnau
Professor K.F. Gauss Matthew Boston
Emmy Germain Emily Allyn Barth
Sergei Langer Mike Mihm
Barry Bell Matthew Archambault

Operators:

Professor K.F. Gauss Jennifer Granville
Emmy Germain Jessica Manley
Sergei Langer Michael Spencer

Musicians:

Roland Synthesizer Robert Schneider
Cello Heather McIntosh
Clarinet Alex Kontorovich

Music composed by Robert Schneider
Performance designed by Michael Spencer
Written and produced by ... Jennifer and Andrew Granville
Directed by Jennifer Granville and Michael Spencer

With the kind assistance of:

*The Institute for Advanced Study,
The Clay Mathematics Institute,
and
The Number Theory Foundation*

The Mathematics of “MSI: THE ANATOMY OF INTEGERS AND PERMUTATIONS”

We begin, as in any mathematical paper, with definitions:

Anatomy (a-nat-o-my) noun: *The scientific study of the shape and structure of an organism and the inter-relation of its various parts. The art of separating the parts of an organism in order to ascertain their position, relations, structure, and function.*

Forensic (fo-ren-sic) adjective: *Relating to the use of science or technology in the investigation and establishment of facts or evidence.*

If you switch on your TV in the evening then, as likely as not, you will find yourself watching an episode of a popular detective show (set in various spectacular locations) in which surprisingly dapper forensic scientists turn up evidence using careful anatomical (and other) study so as to be able to identify and prosecute a heinous criminal. Sometimes a flatfooted detective is misled by the surface evidence to suspect one person, but then the forensic team, digging deeper, turns up details that surprise not only the easily misled detective but even you, the astute viewer. For example, two seemingly unrelated corpses are found, and our hapless detective believes that the crimes are unrelated, whereas the forensic investigators turn up conclusive proof that the two corpses were in fact twins.

So what would happen if we put together a forensic team to investigate the anatomy of some of the most common mathematical objects, say of integers¹ and permutations²? How would they analyze the constituent parts? Could they find a “forensic” way to compare them? And, if so, what conclusions could they possibly draw? Could they find enough uncontested evidence to really “solve the crime”.

Our script “**Math Sciences Investigation: *The Anatomy of Integers and Permutations***” was born of a desire to take this popular approach to discussing the fascinating and extraordinary similarities between the fine details of the structure of integers and of permutations, that is the anatomy of the two, and to write an expository paper that “forensically” analyzes the two. In our script we develop the connection with anatomy and forensics to create a fantasy world where forensic detectives (loosely based on certain famous mathematicians) prove and interpret several of the key notions, eventually discovering some of the deep theorems of the subject. Our goal has been to write a dramatic screenplay that also popularizes mathematical ideas. Moreover we have taken this opportunity to draw attention to several key cultural issues in mathematics:

- How research is done, particularly the roles of student and adviser;
- The role of women in mathematics today;
- The influence and conflict of deep and rigid abstraction.

We have chosen to base our characters on famous mathematicians, exaggerating certain traits. These are not supposed to be accurate biographical portrayals. In this program we discuss the biographies of these luminaries, and how they have motivated our characters. We make no apology for the fact that our perceptions cloud these biographies, just as in any historical discussion.

¹**Integer** (in-te-ger) (noun): *positive or negative whole number.*

²**Permutation** (per-mu-ta-tion) (noun): *a re-arrangement of the elements of a set.*

FAMOUS MATHEMATICIANS: BIOS AND BIAS

K.F. GAUSS

Archimedes, Newton and Gauss, these three, are in a class by themselves among the great mathematicians.

— E.T. Bell

Many central themes of modern mathematics, both pure and applied, came into their modern incarnation through the works of Karl Friedrich Gauss (1777-1855). Gauss's genius was spotted when he was a little boy – At age seven, his teacher told his class to add up all the whole numbers up to 100. Gauss ingeniously determined the correct answer, 5050, in a few moments. By the time he was 16 he had proved several deep mathematical theorems, and had brilliantly interpreted data on prime numbers so as to make a guess as to their distribution, an insight which has motivated research ever since. At 23 he solved the ancient problem as to what regular shapes could be constructed using only a ruler and compass, and by 24 he had written his first book *Disquisitiones Arithmeticae* which is still the model for all introductory books in number theory. Gauss went on to make discoveries in many different areas, notably astronomy and co-creating the telegraph.

Gauss did not publish much, preferring “few but ripe”. Indeed, other mathematicians re-discovered his unpublished work on more than one occasion. He worked well motivating students such as Friedrich Bessel, Richard Dedekind, Sophie Germain and Bernhard Riemann, and his junior colleague, Lejeune Dirichlet:

... usually he sat in a comfortable attitude, looking down ... with his hands folded above his lap. He spoke quite freely, very clearly, simply and plainly ... When he wanted to emphasize a new viewpoint ... he lifted his head ... and gazed ... with his beautiful, penetrating blue eyes while speaking emphatically. ... If he proceeded from an explanation of principles to the development of mathematical formulas, then he got up, and in a stately very upright posture he wrote on a blackboard ... in his peculiarly beautiful handwriting: he always succeeded through economy and deliberate arrangement in making do with a rather small space. For numerical examples ... he brought along the requisite data on little slips of paper.

— Dedekind on Gauss

In our story Professor Gauss had a similar startling childhood. One of his earliest passions were Sherlock Holmes' stories (like many future mathematicians today) which is central to his teaching persona today. Our Gauss is a lover of fine things, his clothes, his art collection, even the furniture in his office. He has been called the “second academic celebrity”,³ with his natural ability to do deep scientific work while explaining it in an accessible manner. This has put him at the forefront of several academic disciplines, while his inventions mean that he is wealthy, and frequently in the public limelight. He is tall, dark and handsome with a confidence in his ability that gives him an attractive aura of being ‘easy in his skin’. Women want him but he projects the sense of being kind

³Albert Einstein was of course the first.

but unavailable. In his mid-50s, his own creativity is waning, but he identifies the most talented students and helps their development, judging well what motivation they need.

EMMY GERMAIN

There are not enough women in research mathematics, even today, though things have been slowly improving in recent years. Although blatant sexism has largely been eliminated, there are still many obstacles to women succeeding in this profession, even with widespread positive discrimination. In the last fifteen years we have seen female undergraduates flourish in North America, increasing from 40% to 60% of the student population, as society's attitudes reform, with an increased expectation that women should enjoy the full opportunities of higher education. It is perhaps changing societal attitudes that are most likely to redress the ridiculous imbalance in our subject, supported by accessible opportunities.

There have been many great female mathematicians in history. From Hypatia in ancient Alexandria, to Sophie Germain in early 19th century France, to Emmy Noether in early 20th century Germany, Julia Robinson, arguably the leading figure in solving Hilbert's 10th problem forty years ago, on to many leading mathematicians today.

Our heroine is a young female mathematician, with an amalgam of the names Emmy Noether (1882–1935) and Sophie Germain (1776–1831); her story represents some of the issues that women have faced in our profession over the last few years. Her background is based on that of Sophie Germain: Germain never formally attended any school or gained a degree. She taught herself geometry, algebra, and calculus, and then Latin and Greek to study Euler and Newton in the original. She was home-schooled! Germain began sending her ideas to Lagrange, and then to Gauss, using the pseudonym “Monsieur Le Blanc”. Joseph-Louis Lagrange demanded to meet with this extraordinary correspondent, and was shocked to find that she was not a “Monsieur”. In 1806, when Napoleon's forces invaded Prussia, Germain asked a personal friend, General Pernety, to make sure of Gauss's safety. It was only then that Gauss discovered her secret, and wrote to her:

But how to describe to you my admiration and astonishment at seeing my esteemed correspondent, Monsieur Le Blanc, metamorphose himself into this illustrious personage who gives such a brilliant example of what I would find it difficult to believe. A taste for the abstract sciences in general and above all the mysteries of numbers is excessively rare: one is not astonished at it: the enchanting charms of this sublime science reveal only to those who have the courage to go deeply into it. But when a person of the sex which, according to our customs and prejudices, must encounter infinitely more difficulties than men to familiarize herself with these thorny researches, succeeds nevertheless in surmounting these obstacles and penetrating the most obscure parts of them, then without doubt she must have the noblest courage, quite extraordinary talents and superior genius. Indeed nothing could prove to me in so flattering and less equivocal manner that the attractions of this science, which has enriched my life with so many joys, are not chimerical, [than] the predilection with which you have honored it.

— Gauss's letter to Sophie Germain

Emmy Noether was also strongly supported from an early age by leading male mathematicians, in her case David Hilbert and Felix Klein. Unfortunately her appointment to the faculty at the University of Göttingen was initially blocked by the philologists and historians, so when she taught, the classes had to be advertised under Hilbert's name. After the First World War she obtained a full faculty position, only relinquishing that when the Nazis banned Jewish faculty in the early 1930s. In 1934, Noether visited the Institute for Advanced Study, although she was not welcome at Princeton University, calling it “the men's university, where nothing female is admitted”. She finished her career on the faculty at Bryn Mawr.

So how should we portray our Emmy? We want real women in our profession who do not need to be so different to belong, and do not need to hide who they are. This is the fantasy world of the movies, so our Emmy is “dark haired and too beautiful to be in a class that promises to cut up cadavers”, why not? Appearances are not everything. In our piece, she confronts sexism, but also receives tremendous support from the people who matter. And when she gets on the right track, there is no stopping her...

DETECTIVE JACK (J.J.) VON NEUMANN

John von Neumann (1903-1957) was a major figure in twentieth century mathematics, and in applications such as the development of the H-bomb, as well as of the modern computer. He had an enormous impact on many fields of pure mathematics as well as developing the ideas behind important applications of mathematics. He was an inspiration to many mathematicians of his time with his ability to penetrate different fields with new perspectives.

von Neumann was another child prodigy: At the age of six, he was able to exchange jokes with his father in classical Greek, and would entertain family by quickly memorizing random pages of the Budapest phone directory. Despite von Neumann's brilliance at school in mathematics, his dad did not want him studying a subject that would not bring him wealth, so they compromised on chemistry. Von Neumann nonetheless attended math classes:

Johnny was the only student I was ever afraid of. If in the course of a lecture I stated an unsolved problem, the chances were he'd come to me as soon as the lecture was over, with the complete solution in a few scribbles on a slip of paper.
— George Polya on von Neumann

In the same year that he got his diploma in chemical engineering, he also obtained his doctorate in mathematics, developing the theory of “ordinals”, an understanding of the different possible types of infinities. For the next three years, von Neumann taught and researched in Berlin. Besides his prodigious mathematical output, he was a denizen of the Cabaret-era Berlin nightlife circuit, enjoying the clubs and the parties.

In 1930 von Neumann came to Princeton, and in 1933, was one of the original six professors at the Institute for Advanced Study.⁴ He worked well with others, indeed

⁴Along with Einstein, Hermann Weyl, Oswald Veblen, Marston Morse and James Alexander.

he could present deep mathematics with amazing lucidity, but was not so good in the classroom; he was notorious for writing complicated equations on the blackboard and erasing them before students had a chance to copy them down!

After marrying in 1938, Von Neumann continued to enjoy good company and parties, but now the parties centered around their house — frequent, famous, and long! Von Neumann was always a snappy dresser and well groomed, the antithesis of the scruffy mathematician. He had as lively views on international politics and practical affairs as on mathematical problems.

In our story Detective Jack (J.J.) Newman is an all-American. Son of hard-working immigrants, he looks and sounds like a native New Yorker. He excelled at school, but was also extremely popular with the other kids, because of his wit and his willingness to party. When we meet him he is 50, well-dressed, though with the look of a man who has had too much good wine and enjoyed the company of many women. He is still somewhat charming, but seems bone-weary, perhaps from too many dead bodies. He has always been resourceful in solving mysteries, and is the only person in the police department who uses the services of academics such as Gauss and Ten-Dieck. At times he is quite brilliant and has even earned the grudging respect of Gauss, who otherwise tends to look down at the investigative skills of police officers. Although under pressure from his captain not to consult Gauss, “an amateur”, he continues to do so in the most intractable cases.

JOE TEN DIECK AND COUNT NICHOLAS BOURBAKI

Alexander Grothendieck (1928–??) is the most controversial mathematician of the second half of the twentieth century. Grothendieck took an abstract, broad perspective on mathematics in a way that paid extraordinary dividends. He took the view that one should only work with very general methods, and only derive concrete results that follow obviously, never allowing oneself to stretch far from the abstract to deduce the tangible. Most mathematicians find it difficult to think this way. Gauss, on the other hand, used poignant examples to motivate most of his theoretical discoveries.

Grothendieck has many ardent followers, some of whom have an almost religious adherence to this way of looking at things, refusing to acknowledge that other techniques could yield anything interesting.⁵ Grothendieck’s point-of-view fit well with the French Bourbaki movement, whose participants sought to rewrite everything in mathematics from an abstract perspective, all “in the correct order”. They would write their books collaboratively — one person would write a chapter and then give it up to the next person to edit, removing any hint of joy or personality from the writing. Given that no one person was supposed to dominate the writing of any one book, they decided to publish all of their output under the collective name, Count Nicholas Bourbaki.

The malign influence of Bourbaki has caused rifts in the research and teaching of mathematics. Too often leaders of the Bourbaki school have refused to acknowledge the top quality work of non-Bourbakiists. Moreover the idea that one must learn mathematics in

⁵Indeed if, say, combinatorics resolves what had been considered an interesting question, then resolute adherents would infer that the original interest must have been a misperception.

such a rigid way has diminished the appeal of our subject to many fine young minds. It is only recently that the Bourbaki influence is diminishing – a sure sign is the welcome rise of combinatorics at the Institute for Advanced Study.

Groethendieck was not well-equipped to minister to his following, dropping out of full-time mathematics in the late 1980s. In 1984 he had proposed a scientific program, that included a very simple type of graph to study Riemann surfaces and the absolute Galois group, which he called a “Child’s Drawing”. Since 1991, he has been in hiding, completely breaking ties with other mathematicians. It is rumoured that he is living as a farmer, in or near the Pyrenees, writing long political and personal tracts.

In our story, Groethendieck is the clairvoyant Joe Ten-Dick, who has some of the personal history and issues of his real counterpart. Any true mathematician, like Gauss and von Neumann, respects the great work of Groethendieck, but is wary of its influence and interpretation, and so it is in our story. Moreover there is one true follower of the cult of Joe Ten-Dick

SERGEI LANGER

Serge Lang (1927-2005) was the most prolific author of research level textbooks in mathematics of the second half of the twentieth century (writing more than 60 books!). Despite the fact that they could be carelessly written and somewhat unmotivated, the sheer volume and the fact that he often wrote the first textbook in a new and exciting area, meant that he had an enormous influence on the development of mathematics. Lang said that the best way to learn a topic was to write a book on it, and he did so quickly; someone once called the Yale math dept to speak to Lang and was told by the receptionist: “He can’t talk to you right now, he is busy writing a book. Can I put you on hold?”

In 1939 Lang’s family fled France for Los Angeles, where he graduated from CalTech in physics at age 19. Moving to Princeton to do a doctorate in philosophy he fell under the influence of Emil Artin (and the new developments in algebra), gaining a PhD in mathematics in 1951. By 1955 he was on the faculty at Columbia, resigning in 1971 in protest of their treatment of anti-war protestors. In 1972 he found a position at Yale, where he settled down and became a prolific author, and tireless campaigner on many issues.

He was a great believer in putting mathematics in a more abstract context, in the Bourbakiist mold of trying to give the big picture, no matter whether the material was suited to that or not:

When I first saw Lang’s Diophantine geometry I was disgusted with the way in which my own contributions to the subject had been disfigured and made unintelligible ... The whole style of the author contradicts the sense for simplicity and honesty which we admire in the works of the masters in number theory — Lagrange, Gauss,... Now Lang has published another book on algebraic numbers which, in my opinion, is still worse than the former one. I see a pig broken into a beautiful garden and rooting up all flowers and trees ... I am afraid that mathematics will perish ... if the present trend for senseless abstraction ... cannot be blocked up. — C.L. Siegel on Lang’s writing (1962)

Lang fought many vigorous campaigns against perceived injustices. He kept copious files on everything that bothered him, in mathematics, in other sciences and in politics, which he would happily share! He had a particular disgust of shoddy use of mathematics in social science research:

I don't like the nonsense that passes for rational discourse so often in our society. I am very much bothered by the inaccuracies, ambiguities, code words, slogans, catch phrases, public relation devices, sweeping generalizations, and stereotypes, which are used (consciously or otherwise) to influence people. I am bothered by the inability of many to recognize these for what they are.

I am bothered by the misinformation which can get disseminated uncritically through the media and by the obstructions which prevent correct information from being disseminated. These obstructions come about in many ways - personal, institutional, through self-imposed inhibitions, through external inhibitions, through outright dishonesty, through incompetence ... I am bothered by the way misinformation, disguised as scholarship, is used in social, political, and educational contexts to affect policy decisions. I am bothered by the way misinformation is accepted uncritically, and by the way people are unable to recognize it or reject it.

— Serge Lang

In our story time has telescoped and Sergei Langer is part of the new wave of immigration to the US following the collapse of the Soviet block. He had a formal training back in Prussia where, coming from the upper classes, he has a dueling scar across his right cheek, shaped like an integral sign.⁶ He is blond, muscular, and narcissistic. Although wrinkled, from a distance he can be mistaken for someone much younger!

BARRY BELL

E.T. Bell (1883-1960) was the greatest of all mathematical biographers. In “Men of Mathematics”, Bell crafted romantic histories of the great mathematicians of the 17th, 18th and 19th centuries, inspiring many generations of prodigies to see a bright future in the subject. Mathematically the book is fairly accurate, though some of the life stories are ambitious reconstructions based on little documentary evidence.

There are few mathematical biographers today, most notably Constance Reid, and Siobhan Roberts (who is a Director’s Visitor to the Institute this year). There are several mainstream reporters of mathematics today — Ivars Peterson, Barry Cipra, Dana Mackenzie, Keith Devlin,...; our reporter’s name is an All-American amalgam of Barry Cipra and E.T. Bell.

⁶You’ve guessed it, we could not resist some reference to Harry Potter!

BIOGRAPHIES OF THE PERFORMERS

Andrew Granville is the Canadian Research Chair in number theory at the Université de Montréal, and a member of the School of Mathematics at the Institute. His current research, working with Soundararajan, is focused on a “pretentious” understanding of analytic number theory. He is the author of over one hundred research papers, and more than a dozen expository articles. He has received various research and writing awards, including the 2008 Chauvenet prize for the exposition in *It is easy to determine whether a given integer is prime*. His only acting experience was as Alan Turing in the Institute Playreaders’ 1990 production of *Breaking the Code*.

Having written quite a few expository articles, I have become interested in going further to develop new and more effective ways of communicating what math has to offer to a wide audience. It has been quite a challenge to co-write MSI: Anatomy, developing the drama, while at the same time giving enough flavour to the mathematics to be able to claim that this is also “expository writing”.

Jennifer Granville is the co-director of the Northern Film School at Leeds Metropolitan University in England, and has worked as an actor, playwright, script editor, and producer. She began as an actor, appearing in repertory theatre across the UK, and in films such as *Loophole* (1981, with Martin Sheen and Albert Finney), *Kean* (1978, with Anthony Hopkins), and as Mrs. Hurst in the BBC TV series *Pride and Prejudice* (1980). Her recent one woman play, *The Rule of Three*, has been performed on both sides of the Atlantic. As a screenplay writer, she has won the La Femme, 21st Century and Latino Competitions. Her screenplay *Rumpringa*, co-written with Lucy Walker, is now being produced by Arcane Pictures.

She produced the 16mm short *The secret songs of Butterfish*, which aired on television internationally and won the Gold Plaque in Chicago and the Special Jury Award at the New York Expo. Most recently she produced *Winesburg, Ohio* adapted from the novel by Sherwood Anderson, and *Lost in Winesburg*, a feature length documentary.

I am interested in allowing actors, writers and directors more artistic freedom to collaborate than in their traditional roles, in the making of a play or film. There is always collaboration across many disciplines: Can that be made to be more creative than technical? In co-writing and producing MSI: Anatomy we have explored a non-traditional collaboration, in which participants have had the freedom to integrate their creative, as well as technical, strengths.

Michael Spencer’s design career features a long collaboration with director Andrew Manley (most recently at Colorado College in the US) and an infamous *Marriage of Figaro* for the Welsh National Opera. In 1991 he became the first person in the UK to be awarded an MA in the subject of Theatre Design. He is currently Course Director of what is now the BA Performance Design & Practice course at Central Saint Martins College of Art & Design in London, a course with a distinguished heritage in the field of theatre design, reconfigured to reflect an expanding field.

I was good at two things at school: Maths and Art, a combination that left most of my teachers confused as to how I could put these abilities to use. For me, creating any artwork and revealing hidden mathematical properties have everything in common in terms of process. I wanted to see if it were possible to provide a visual metaphor that would support and even illuminate the mathematics in MSI: Anatomy — as well as entertain of course.

Robert Schneider (Roland JX-3P analog synthesizer) is a composer, musician, and record producer, best known as lead singer of the pop group *The Apples in Stereo*, and as a co-founder of *The Elephant 6* Recording Co., an influential, and often experimental, collective of musicians and artists. He has performed around the world, produced critically acclaimed albums, appeared on late night TV (including *The Colbert Report*) and in a major motion picture (Mike Myers' *The Love Guru*), composed advertising jingles, released children's music under the alias *Robbert Bobbert*, received the 2008 Independent Music Award for best Pop/Rock Song *Same Old Drag*, and was surprised to hear the band's hit *Energy* performed by the contestants on *American Idol*.

One of my "side projects" is to study mathematics at the University of Kentucky. I have given an invited MAA lecture at the 2008 annual meeting on a musical scale I invented based on logarithms, presented my research in number theory at the student session, and hosted a math-themed film festival at the 2009 joint meeting.

Jessica Manley (Emmy Germain's operator) has appeared in regional theatres all over the UK since childhood, and co-starred (with Ben Kingsley) as Margot Frank in the ABC TV mini-series *Anne Frank: The Whole Story*. After training at the Guildhall School of Music and Drama in London, she played Margaret in *Man for All Seasons*, and Miranda in *The Tempest* for TARA Arts Theatre. TV includes the BBC's first interactive internet drama *Signs of Life*. Film credits include *Miriam* in the prize-winning short films *Veils* and, most recently, as a Gypsy woman being rescued by Benicio Del Toro in the soon-to-be released *Wolfman*. Jessica is the Artistic Director of LISTED THEATRE, a site-specific theatre company, combining art and activism to reawaken spaces and places of historical significance.

Lorraine Wochna (Narrator) is a reference and instruction librarian at Ohio University, who works closely with the Department of English, and the Schools of Film and Theatre. She has her MLS in Library Science, MA in Theatre Criticism and is working on her MA in film studies. Her thesis research is on the theory of phenomenology and audience focusing on The Wooster Group's production of *Hamlet*. She lived in NYC for 20 years performing, producing and word processing. She is an avid supporter of all things new.

Carl Wallnau (Jack Von Neumann) is the Artistic Director of CENTENARY STAGE COMPANY, and chairman of Communication & Fine Arts Department at Centenary College. Recent acting credits include Emperor Joseph in *Lorenzo* at the New York Musical Theatre Festival, and Andrew Maddock in the world premiere of *Any Other Name* at Premiere Stages. He has appeared at the Actors Playhouse in NYC, at numerous regional theatres,

and was 14 months on the first national tour of *Titanic*. He has also directed numerous productions including the world premieres of *Inventing Montana*, *The Tillie Project* and *The Poetry of Pizza*. He received an MFA in Acting from Rutgers University.

Matthew Boston (Professor K.F. Gauss) appears frequently in regional theatre including *The Diary of Anne Frank* at the Intiman Theatre, *Black Snow* at Yale Rep, as well as *Arcadia*, *The Cherry Orchard*, *Mrs. Warren's Profession* and *The Matchmaker* at the American Conservatory Theater; and in NYC, including *Disconnect* at the Working Theater, and *Magic Hands Freddy* at the SoHo Playhouse. Film and TV credits include *Law & Order*, *Ghost Ship*, *Into the Blue*, *Camp Wilderness*, *One Life to Live* and *All My Children*.

Emily Allyn Barth (Emmy Germain) began acting professionally before she turned five, working in regional theatre, commercials, and film. She has performed all over the world, including at the Edinburgh Fringe Festival in Scotland, in the American production of *Ghetto* directed by Israeli playwright Joshua Sobol, and for President and Mrs. Clinton at The White House. She is currently starring in the web sitcom *Chaos Theory*, for which she is also a writer, and hosting the *No Clue Movie Review*. Television credits include Mary the diner waitress on *One Life to Live*, *Days of our Lives*, *Saturday Night Live* and *The Education of Max Bickford*. Emily holds a BA in Film Studies from Wesleyan University.

Mike Mihm (Sergei Langer)'s theatre credits include *Camino Real* at Provincetown Tennessee Williams Festival, *From Russia With Angst* with Workshop Theater, multiple installations of *The Dare Project* with Tax Deductible Theatre, *Shakespeare in ACTION* and *The Birthmark* with Stages on the Sound, and *Mister Roberts*, *A Christmas Carol*, and *The Triumph of Love* while interning at Cincinnati Playhouse. Films include: *The Taking of Pelham 123*, *The Dying Western*, and *Killing the Joneses*.

Matthew Archambault (Barry Bell) has worked with various theatre companies including playing Mark Anthony in *Julius Caesar* at the Roxy Regional Theatre, Gooper in *Cat on a Hot Tin Roof* at the Monomoy Theatre, and several roles at the Virginia Shakespeare Festival, and with the Brooklyn Stage Company. His directing credits include *Closer* and *Sex: aka Wieners and Boobs*. Matt received his MFA in acting from the University of Houston under Jack Young. Visit him online at matthewarchambault.net.

Heather McIntosh (cello) is a cellist, composer, and recording artist from Athens, Georgia, who leads the musical project *The Instruments*, composes film scores, such as for the documentary *Examined Life*, Astra Taylor's portrait of modern philosophers, and performs with numerous well-known groups, including *The Circulatory System* from the *Elephant 6 collective*, and R&B artists Gnarls Barkley and Lil Wayne. She is also a curator of AUX, an organization devoted to experimental music.

Alex Kontorovich (clarinet) is a clarinetist and saxophonist who has toured around the world playing with the Grammy Award-winning Klezmatics and the Klez Dispensers. Kontorovich has performed at venues such as the Montreal International Jazz Festival, the

Lincoln Center, and the Royal National Theatre in London. His 2007 album *Deep Minor* was called “an exuberant if not groundbreaking slice of downtown klez-jazz” by TimeOut, NY. He was recently featured on the cover of *Saxophone* magazine. Alex is a Tamarkin Assistant Professor of mathematics and an NSF postdoctoral fellow at Brown University and is currently a member of the School of Mathematics at the Institute.

REVERIE IN PRIME TIME SIGNATURES

composed by *Robert Schneider*

As the title indicates, the piece is written in prime-numbered time signatures, which is to say, there is a prime number of beats in each measure. The main theme plays in the time signature $7/4$, which indicates 7 beats per measure, with an interlude that passes through the signatures $2/4$, $3/4$, and $5/4$ as well. From the constraints imposed by these rhythmic patterns, melodies emerged naturally as I composed, special to each prime.

A second interlude happens in $29/4$ time, occurring, by a pleasing coincidence, at the 29th measure of the composition, a musical rendition of the sieve of Eratosthenes, an ancient Greek method for identifying prime numbers.

Here, a high note pulses on every beat, rising in pitch at the perfect squares; while the cello plays a note on every other beat, the clarinet every third beat, and the keyboard plays a rich chord on every fifth beat, that every fifth beat is marked by a chord instead of a single note, is intended as a nod to the golden ratio, which is related to the square root of 5, and has historically been considered a model of aesthetic perfection by some writers.

Notice how the cello marks beats that are multiples of 2, the clarinet marks multiples of 3, and the chords mark multiples of 5. Clearly, the beats on which none of these instruments play must not be multiples of 2, 3 or 5, which is enough to identify them as primes among the integers relevant to the composition, each accompanied by only the high pulse; until the cello, clarinet, and keyboard chords sound together on the 30th beat, resolving before returning to the main theme.

In this tangled interlude, not quite random, our ears experience the formation of the sequence of the primes.

I have read that Leonardo Da Vinci may have hidden a musical composition in his painting *The Last Supper*, and that the Roslyn Chapel in Scotland has musical notation encoded in the masonry. As a variation on this theme, I sought to encode a hint of real mathematics within the musical composition: Eratosthenes’ first step toward understanding the primes.

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Wolfensohn Hall, The Institute for Advanced Study, Princeton.