

Concours Putnam

Atelier de Pratique

Le mardi, 26 septembre 12h30-13h30

5448 Pav. André Aisenstadt

1. Knowing that the expression

$$\sqrt{3 + \sqrt{3 + \sqrt{3 + \dots}}}$$

converges, find its value.

2. Let $P(x)$ be a polynomial of degree n satisfying $P(k) = k$ for $k = 1, \dots, n$ and $P(0) = 1$. Find $P(-1)$.
3. A car dealership that was open 7 days a week sold at least one car each day in 2015 and a total of 600 cars during that year. Prove that there was a period of consecutive days during which exactly 129 cars were sold. (There were 365 days in 2015).
4. A car travels from one city to another at a rate of 40 miles per hour and then returns at a rate of 60 miles per hour. What is the average rate for the round trip?
5. Transportania is a country with finitely many cities, each of which is directly connected by a road with exactly three other cities. Thus, a traveler who arrives at a city along one of the three roads leading into it can choose between the two other roads, one to his left and one to his right, to continue his trip, assuming that he does not want to return to the city he just came from. Suppose that a traveler starts at city A , goes to city B , there takes the road to his right to city C , then takes the road to his left to city D , and so on, alternating between the left and the right road. Prove that he eventually gets back to city A .