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Geometric Group Theory, Hyperbolic Dynamics and Symplectic Geometry

Organised by
Gerhard Knieper, Bochum
Leonid Polterovich, Tel Aviv/Chicago
Leonid Potyagailo, Villeneuve d'Ascq.

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Abstract. Invariants of topological spaces of dimension three play a major role in many areas, in particular ...

Mathematics Subject Classification (2000): AMS-CLASSIFICATION.

Introduction by the Organisers

Organisers start here...

Workshop: Geometric Group Theory, Hyperbolic Dynamics and Symplectic Geometry

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Abstracts

Towards categorification of Lagrangian Topology

Octav Cornea
(joint work with Paul Biran (ETH))

Cobordism has played fundamental role in the modern development of algebraic and differential topology. In symplectic topology, Lagrangian cobordism has been introduced by Arnold at the beginnings of the field and it has been studied by Eliashberg and Audin who showed that without any additional restrictions it is a very flexible notion. Later on in the middle of the '90's Chekanov remarked that if additional restrictions are imposed - for instance monotonicity - then some rigidity is present. In the paper [1] - which was the main reference for the talk - we consider Lagrangian cobordism from a more categorical point of view: we first notice that it is possible to define a Lagrangian cobordism category whose objects are the Lagrangian submanifolds of a given symplectic manifold (M, ω). The morphisms between two such objects L, L' are horizontal isotopy classes of Lagrangian submanifolds $V \subset (\mathbb{C} \times M, \omega_0 + \omega)$ so that V is non-compact and has one positive end that is identified with $[0, 1) \times \{1\} \times L$ as well as some negative ends identified with $(-\infty, 0] \times \{1\} \times L_1, \ldots, (-\infty, 0] \times \{k\} \times L_k, (-\infty, 0] \times \{k+1\} \times L'$ for some $k \geq 0$. It is not difficult to show that this does indeed give rise to a category that we denote by $\mathcal{C}ob_{pre}(M)$.

Remark. Another category of Lagrangian cobordisms has been introduced by Nadler and Tanaka also in an October 2011 preprint.

From now on restrict to the subcategory $\mathcal{C}ob^d_{pre}(M)$ of all Lagrangians that are uniformly monotone in the sense that the Maslov morphism and the symplectic area are proportional with the same constant, the minimal Maslov number is at least 2 and additionally the number of J-holomorphic disks through a point is the same for all Lagrangians (+ a condition having to do with the appropriate Novikov ring). The morphisms in this subcategory also satisfy the same conditions. Denote by $DFuk^d(M)$ the derived Fukaya category with the same objects as those of $\mathcal{C}ob^d_{pre}(M)$. The main result is that there exists a functor:

$$\mathcal{F} : \mathcal{C}ob^d_{pre}(M) \to DFuk^d(M)$$

that is the identity on objects and that fits, in an appropriate sense with the triangulated structure of the target. For instance, given a cobordism V as above this compatibility implies that, in $DFuk^d(M)$, L belongs to the subcategory generated by $L_1, L_2, \ldots L_k, L'$. In fact, the construction provides exact triangles in $DFuk^d(M)$: $L_2 \to L_1 \to M_2, \ldots, L_{i+1} \to M_i \to M_{i+1}$ (with $L' = L_{k+1}$) and $M_{k+1} \simeq L$.

References

[1] P. Biran, O. Cornea, *Lagrangian cobordism I*, preprint Fall 2011, arXiv:1109.4984.

Reporter: Kamil Bieder