Mortality at advanced ages

Louis G. Doray, Ph.D., A.S.A.

Department of Mathematics and Statistics, University of Montreal

doray@dms.umontreal.ca

North American actuaries have used Gompertz and Makeham models for mortality at advanced ages for over a century. The force of mortality increases without bound with age. With recent progresses in medicine, more people have been surviving to extreme ages and the percentage of oldest old peole has been increasing very rapidly.

Statistics Canada reports that in 1971, in Canada, there were 13940 males and 23440 females aged 90 and over; in 1991, those numbers were 25055 males and 68795 females in that age group. Over the course of only one generation, the increase is phenomenal: more than three times more males and close to 6 times more females aged 90+ in 1991 than in 1961.

With this growing population of very old people, will come an increase in the total pensions payable from the Canada/Quebec Pension Plan (pensions will be payable to more people who survive for longer periods) and also an increase in health costs. Actuaries will need to take into account this increasing longevity when they price insurance products or value annuities for pension plans.

In the 20th century, actuaries in England and demographers have been considering alternate models besides Gompertz and Makeham's, where the force of mortality increases up to around age 80, then starts levelling off and is bounded as the age goes to infinity.

In 1932, Perks developed the logistic model in which the force of mortality

at age x is given by the 4-parameter function

$$\mu_x = \frac{A + Be^{\mu x}}{1 + Ce^{\mu x}}.$$

The Gompertz ($A = 0,\ C = 0$) and Makeham ($C = 0$) models are special cases of the logistic model. This is useful to test one model versus another one when fitting them to population data. The logistic model can arise in a heterogeneous population where each member has a Makeham force of mortality and where the parameter B varies among individuals according to a gamma distribution. This Makeham-gamma model is a frailty model.

The logistic model has been successfully fit to mortality data of aged people in 13 industrialized countries in Europe and Asia for the periods 1960-70, 1970-80, 1980-1990 and for the cohort born in 1871-1880. To evaluate the 4 parameters of the model with Canadian data, we need the number of deaths and of people at each age above 80.

From 1949 to 1997, Statistics Canada has published the annual number of deaths in Canada by sex and individual ages from 0 to 99 and grouped over age 100. But the number of people alive at individual ages is not known and must be inferred from the number of deaths using the method of extinct generations. We can calculate the number of people of age x living in a year by summing the number of deaths in future years at successive ages $x + 1, x + 2, \ldots$. A key assumption of this method is that no migration takes place after age x.

More research needs to be done on those models to see the effect of their use on the projection of the Canadian population.