Math 103. Section 205. Quiz 2b

March 1st, 2007

NAME:

STUDENT NUMBER:

Calculators and other electronic devices are neither allowed nor required for this test.
This exam has 2 pages.

Compute the following integrals:

1. $\displaystyle\int \frac{dx}{x^2+2x-3}$

Solution: $\displaystyle\int \frac{dx}{x^2+2x-3} = \int \frac{dx}{(x-1)(x+3)}$.

We will solve this problem using partial fractions

$$\frac{1}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3} = \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$$

$\begin{cases} A+B=0 \\ 3A-B=1 \end{cases}$. From first equation $A=-B$, adding both equations, $4A=1$.

Therefore, $A=\frac{1}{4}$ and $B=-\frac{1}{4}$.

(We can also solve it by plugging $x=-3$ and $x=1$ in $A(x+3)+B(x-1)=1$.)

$$\int \frac{dx}{x^2+2x-3} = \int \left(\frac{1}{4(x-1)} - \frac{1}{4(x+3)}\right) dx = \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+3| + C$$

2. $\displaystyle\int x\cos(x)dx$

Solution: We solve this problem using integration by parts. Let $u=x$ and $dv=\cos(x)$. Then $du=dx$ and a possible $v=\sin(x)$.

$$\int x\cos(x)dx = x\sin(x) - \int \sin(x)dx = x\sin(x) + \cos(x) + C$$

3. $\displaystyle\int \frac{\ln(2x)}{x}dx$

Solution: We solve this problem by using substitution $u = \ln(2x)$. Then $du = \frac{dx}{x}$.

$$\int \frac{\ln(2x)}{x}dx = \int u du = \frac{u^2}{2} + C = \frac{\ln^2(2x)}{2} + C$$

4. $\displaystyle\int_0^3 \sqrt{9-x^2}dx$ (hint: $x = 3\sin u$, $\displaystyle\int_0^{\frac{\pi}{2}} \cos^2 y\, dy = \frac{\pi}{4}$).

Solution: We do the substitution $x = 3\sin u$. Then $dx = 3\cos u du$

$$\int_0^3 \sqrt{9-x^2}dx = \int_0^{\frac{\pi}{2}} \sqrt{9-9\sin^2 u}\, 3\cos u\, du = \int_0^{\frac{\pi}{2}} 9\cos^2 u\, du = \frac{9\pi}{4}$$

The problem can be also solved by observing that we are computing one fourth of the area of the circle of radius 3.