# THE CONLEY INDEX IN HILBERT SPACES WITH APPLICATIONS

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Titles of talks:

1. The Conley index for flows.
2. The homotopy $\mathcal{LS}$–index and strongly indefinite problems.
3. Cohomology of the $\mathcal{LS}$–index and generalized Morse inequalities.
4. The $\mathcal{LS}$–index in the presence of symmetries.
5. Applications of the $\mathcal{LS}$–index to certain ODE's and PDE's of variational type.

**An outline of lectures:**

A brief introduction to the Conley index theory for flows in a locally compact spaces will be given in the first lecture. It is not our intention to develop the theory rigorously but the aim is to provide an intuitive understanding of the index. A short presentation of preliminary definitions and basic facts in the theory of Conley will be given. Next, a few examples will be discussed to illustrate the most important properties of the index.

Lecture two will be devoted to an extension of the classical Conley index to flows on an infinite–dimensional Hilbert space $H$ generated by vector fields $f : H \to H$, $f(x) = Lx + K(x)$, where $L : H \to H$ is a bounded linear operator satisfying certain technical assumptions and $K$ is a completely continuous perturbation. An example will be discussed to show how this new invariant, called the $\mathcal{LS}$-index, can be applied in searching critical points of strongly indefinite functionals having asymptotically linear gradient.

Subsequently, in lecture three, the $\mathcal{LS}$–index theory will be developed. In particular, the cohomological version of the index will be presented and Morse inequalities for Morse decompositions of isolated invariant sets will be discussed.

Assume we are given a linear and orthogonal action of a compact Lie group $G$ on a Hilbert space $H$. Consider a local flow on $H$ generated by a vector field $f$ as above and such that $Lgz = gLz$ and $K(gz) = gK(z)$ for every $g \in G$ and $z \in H$ ($f$ is a $G$–equivariant vector field). Those kind of flows appear for instance, in problems concerning multiplicity results for periodic solutions of autonomous Hamiltonian systems which admit natural symmetries of the group $S^1$. Also in a nonautonomous case, assuming on a Hamiltonian function $Q : R \times R^{2m} \to R$ to be invariant with respect to an action of a compact Lie group $G$ on $R^{2m}$ one is lead to consider flows generated by $G$–equivariant vector fields. The equivariant Conley index theory provides natural tools for searching flows with symmetries. We will briefly describe that version of the $\mathcal{LS}$–index in lecture four.

Multiplicity problems for periodic solutions of certain types of Hamiltonian systems and for solutions of indefinite elliptic systems will be considered in lecture five. The $\mathcal{LS}$–index will be used as a tool.

**References:**


1. S. Angenent and R. van der Vorst, A superquadratic indefinite elliptic system and its Morse-Conley-Floer homology, Math. Z., 231 (1999) 203–248.

2. K.C. Chang, Infinite Dimensional Morse Theory and Multiple Solution Problem, Birkhäuser,Boston, 1993.

3. C.C. Conley, Isolated Invariant Sets and the Morse Index, CBMS Regional Conf. Ser. in Math. 38, Amer.Math.Soc.,Providence,1978.

4. A. Floer, A refinement of the Conley index and its application to the stability of hyperbolic invariant sets, Erg. Theory and Dyn. Sys., 7 (1987) 93–103.

5. K. Gęba, M. Izydorek, A. Pruszko, The Conley index in Hilbert spaces, Studia Math. 134(3)(1999) 217–233.

6. M. Izydorek A cohomological Conley index in Hilbert spaces and applications to strongly indefinite problems, J. Differential Equations, 170(1) (2001) 22–50.

7. M. Izydorek, Equivariant Conley index in Hilbert spaces and applications to strongly indefinite problems, Nonlinear Anal. T.M.A. 51 (2002) 33–66.

8. Izydorek and K.P. Rybakowski, On the Conley index in Hilbert spaces in the absence of uniqueness, Fund. Math., 171 (2002) 31–52; The Conley index in Hilbert spaces and a problem of Angenent and van der Vorst, Fund. Math., 173 (2002) 77–100; Multiple solutions of indefinite elliptic systems via a Galerkin–type Conley index theory, Fund. Math., 176 (2003) 233–249,

9. K. Mischaikow, Conley Index Theory, Dynamical Systems, Ed., R. Johnson, Lecture Notes in Math. No. 1609, Springer–Verlag, (1994).

10. D. Salamon, Connected Simple Systems and the Conley Index of Isolated Invariant Sets, Trans. AMS No 1, Vol 291 (1985), 1–41.