Kam for the non-linear Schrödinger equation

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Abstract

We will present a recent work (*joint with S. Kuksin* [2]) on the perturbation theory of finite-dimensional KAM-tori (= finite-dimensional invariant tori with a dynamics that is linearizable and reducible) for the non-linear Schrödinger equation in dimension d (with periodic boundary conditions). The difficulties to apply a KAM-approach to this equation are substantial for $d2$, and its "Töplitz-Lipschitz"-property is essential to handle these difficulties.

The existence of quasi-periodic solutions (= finite-dimensional invariant tori with a dynamics that is linearizable) has been proved by Bourgain in 2003 [1]. Our work provides a new proof of the existence of such solutions, a proof that also provides the reducibility of these solutions and, as a consequence, their linear stability.

References

[1] J. Bourgain, *Greens function estimates for lattice Schrödinger operators and applications*, Ann. of Math. Stud., Princeton University Press, Princeton, 2004.

[2] L.H. Eliasson and S.B. Kuksin, KAM *for non-linear Schrödinger equation*, mp-arc 06-134 (2006).