Birkhoff normal form and almost global existence for some Hamiltonian PDEs

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Abstract

In these lectures I will present some recent results of Birkhoff normal form for Hamiltonian PDEs focusing on the nonlinear wave equation

$$u_{tt} - \Delta u + \mu^2 u = u^2$$

on a d dimensional sphere or on a segment with Neumann boundary conditions. The main dynamical consequences of the normal form theorem are a lower estimate of the times of existence of small solutions and a qualitative description of the dynamics.

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