

# Variational method for the problem of Arnold diffusion

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## Abstract

The problem of Arnold diffusion is raised for nearly integrable Hamiltonian systems. It concerns whether there exists an orbit which in the infinite past approaches to a small neighborhood of any given point of action and in the infinite future approaches to a small neighborhood of any another point of action (should be in the same energy level if the system is autonomous). It has been widely accepted to call it diffusion orbit. In the variational language, if there is an orbit connecting two different Aubry sets, the action along this orbit undergoes substantial variation. It is the main object in these lectures.

The lectures consists of four parts. The first part is devoted to an introduction of Mather theory, it includes the conception of minimal measure, Mather set, Aubry set and Mañé set as well as barrier function. The second part concerns the so-called weak-KAM theory, some property of elementary weak-KAM solution can be used to study certain continuity of barrier function. The third part is to show the existence of local connecting orbits between two Aubry sets if their first de Rham cohomology are close to each other, under the condition of some topological triviality of the Mañé set. The last part is for the variational construction of global connecting orbits which connects two different Aubry sets corresponds to two different cohomology classes.

## References

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