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# Geometric approaches to diffusion and instability

R. de la Llave
*Department of Mathematics*
*University of Texas*
*Austin TX 78712-1802*
*USA*
llave@math.utexas.edu

**Abstract**

It has been known since the 60's that many Hamiltonian systems close to integrable are stable for many for all the time, for all the points are stable for large time, but also that they are not stable for all the points for all the time.

The goal of the lectures is to give a geometric characterization of some mechanisms of instability. Other variational approaches are given by Prof. Cheng.

We plan to introduce several geometric objects that have easy to ascertain long term behavior and, then explain how the different interactions between these objects can lead to instability and many other effects.

- Some basic objects:
  - KAM tori
  - Normally hyperbolic manifolds
  - Normally hyperbolic laminations
- Some classical tools:
  - Averaging theory
  - Melnikov theory
- Some more recent tools:
  - The scattering map
  - The separatrix map
  - Correctly aligned windows
- Putting it all together.
  - Some basic examples
  - Estimates on time, Hausdorff dimension
  - Some tentative statistical properties

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