Normal forms of holomorphic dynamical systems

Laurent Stolovitch
Université Paul Sabatier, MIG
Laboratoire de Mathématiques Émile Picard
31062 Toulouse Cedex 09
FRANCE
stolo@picard.ups-tlse.fr

Abstract

We shall present various recent results about normal forms of germs of holomorphic vector fields at a fixed point. We shall explain how relevant it is for geometric as well as for dynamical purpose. We shall first give some examples and counter-examples about holomorphic conjugacy. Then, we shall state and prove a main result concerning the holomorphic conjugacy of a commutative family of germs of holomorphic vector fields. for this, we shall explain the role of diophantine condition and the notion of singular complete integrability. We then sketch a new KAM phenomena (non-symplectic) that occur when considering the perturbation of the previous situation.

References

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