

Zeta and L-Functions of Graphs

Lecture 1. Three Zeta Functions and the Riemann Hypothesis.

The plan is to briefly explore the Riemann, Selberg and Ihara zeta functions. The later is associated to a finite graph. Its Riemann hypothesis is of interest for expander graphs. Most of the lectures come from a manuscript on my website: www.math.ucsd.edu/~aterras/newbook.pdf.

Lecture 2. The Ihara Zeta Function as a Ruelle Zeta Function of a Dynamical System.

The Ruelle zeta came out of the Artin-Mazur zeta associated to a diffeomorphism of a smooth compact manifold, which in turn came from the zeta function of a variety over a finite field. We will see that the Ihara zeta is a special case of the Ruelle zeta and thereby prove a determinant formula. Then we derive the prime number theorem for graphs.

Lecture 3. Artin L-Functions of Graph Coverings.

After defining graph coverings, summarizing their Galois theory, and considering analogs of Artin L-functions we prove the graph theory Chebotarev density theorem.

Lecture 4. Edge and Path Zetas, More Determinant Formulas, Connections with Quantum Chaos.

Several variable analogs of the L-functions of the preceding lectures will be defined. The spacings of the poles and connections with random matrix theory will be considered.