DEFINITIONS AND PREDICTIONS OF INTEGRABILITY FOR DIFFERENCE EQUATIONS

Hietarinta, Jarmo

February 14, 2008

Abstract

The concept of integrability for difference equations. Integrability means, roughly speaking, regularity (and solvability is not integrability, as examples from chaos theory illustrate). Existence of a symmetry in the sense of commuting flows. Existence of a conserved quantity. For continuous systems integrability is often defined as the existence of a (nontrivial) Lax pair from which many results follow, this extends to discrete systems as well. For partial difference equations their consistent extension to higher dimensions is very strong condition and also fruitful as it results with a Lax pair. Algorithmic methods for predicting integrability. The "singularity confinement criterion", which is in some ways comparable to the Painlevé analysis of differential equations. It works well but is not perfect, a more accurate prediction is obtained by computing the algebraic entropy of the iterations, but that is more computationally intensive. Other methods include Nevanlinna growth analysis.